ASTRON 329/429, Fall 2017 – Problem Set 2

Due on Oct. 19, in class.

Solve the following problems, plus problems 5.1, 5.2, 5.5, 5.9, and 5.10 in Ryden:

I. Matter- and radiation-dominated limits. Assume a flat universe ($k = 0$) and consider power-law solutions $a \propto t^q$. Showing all your steps, solve for $a(t)$ and $\rho(t)$

a) in the matter-dominated limit.
b) in the radiation-dominated limit.

Compare the time evolution of ρ in both limits. In which limit does the cosmic energy density decline faster with respect to time?

II. Radiation pressure. Consider a gas of photons moving with random directions and let $\rho_{\rm rad}c^2$ be the energy density of this photon gas. Imagine a fictitious wall in this gas. By evaluating the momentum per unit time per unit area of photons crossing the wall from one side to the other, show that the radiation pressure

$$p_{\rm rad} = \frac{\rho_{\rm rad}c^2}{3}. \tag{1}$$

Note that this result holds for any relativistic species, such as relativistic neutrinos. What is the equation of state parameter w for such a gas?

III. Distances in an arbitrary EOS universe. Suppose that you are in a spatially flat universe containing a single component with a constant equation of state parameter w.

a) Derive expressions for the comoving, luminosity, and angular diameter distances as a function of redshift ($d_{\rm com}(z)$, $d_{\rm lum}(z)$, and $d_{\rm diam}(z)$).

b) At what redshift does $d_{\rm diam}(z)$ peak?

IV. Cosmological constant in Planck units. The Planck system of natural units is based on constants of nature. Show that the following Planck mass, length, and density

$$m_{\rm P} = \left(\frac{\hbar c}{G}\right)^{1/2}; \qquad l_{\rm P} = \left(\frac{\hbar G}{c^3}\right)^{1/2}; \qquad \rho_{\rm P} = \frac{m_{\rm P}}{l_{\rm P}^3} \tag{2}$$

have the expected units of g, cm, and g cm^{-3}, respectively. Although it is not well understood how to compute it correctly, quantum field theory is often assumed to naturally predict a vacuum energy density $\rho_\Lambda \sim \rho_{\rm P}$. For our universe with $H_0 \approx 70$ km/s/Mpc and $\Omega_\Lambda \approx 0.7$, what is the cosmological constant energy density in units of the Planck density, i.e. what is the value of the dimensionless ratio $\rho_\Lambda/\rho_{\rm P}$.

You should find a very small number, $\ll 1$. This large discrepancy between the "natural" value for the vacuum energy density and the observationally inferred value is what is termed the "cosmological constant problem." If ρ_Λ were exactly zero, this would not be considered as big of a problem because it might be assumed that the underlying theory has an exact cancelation (symmetry) that gets rids of the vacuum energy density. However, it is very difficult to understand how positive and negative contributions $\sim \rho_{\rm P}$ to the vacuum energy density would conspire to *almost* cancel each other to the fractional accuracy $\rho_\Lambda/\rho_{\rm P} \ll 1$ but not exactly.